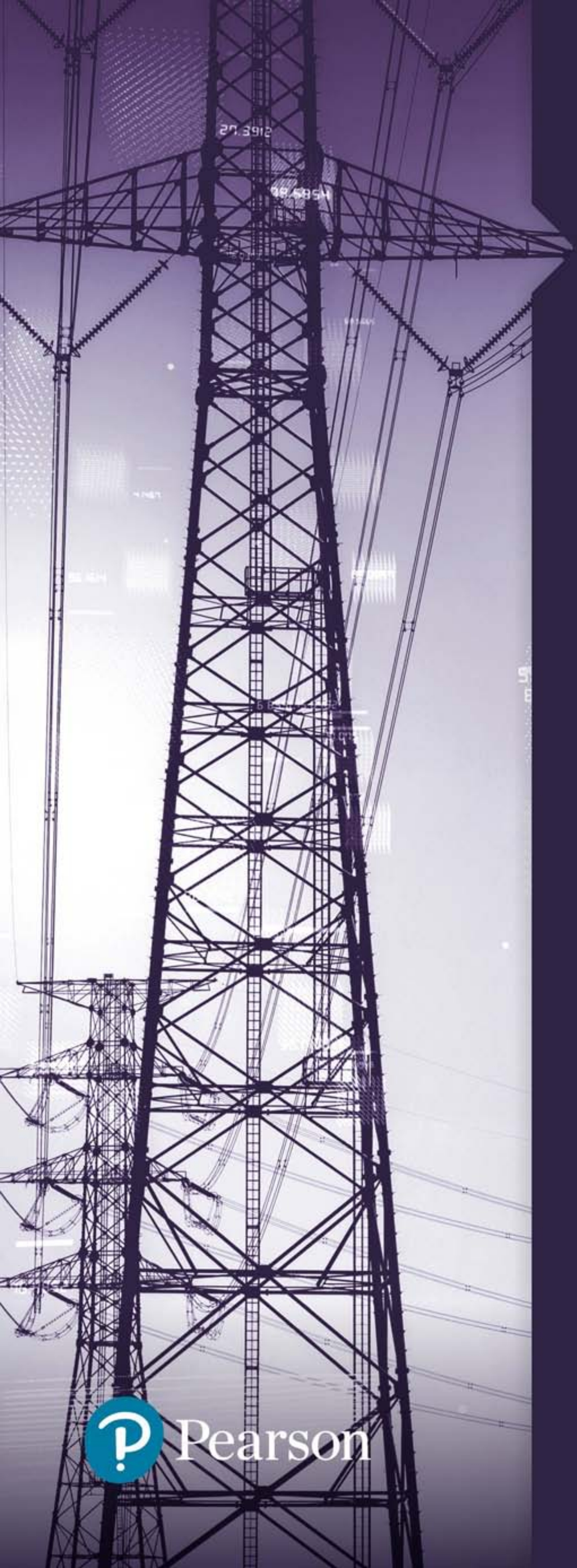

POWER SYSTEM ANALYSIS

A DYNAMIC PERSPECTIVE

K. N. SHUBHANGA

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Power System Analysis

A Dynamic Perspective

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To My Parents,
Teachers and Students

Brief Contents

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Foreword

Although electrical power systems have been in existence for more than a century, challenges in this field abound. While the structure of a bulk power system has remained more or less unchanged, the deployment of emerging technologies like wind and solar energy systems, voltage-source (power electronic) converters, and wide-area measurement systems have had a significant impact on the operation and control of power systems. An abiding concern of a power system engineer has been the stability of the electrical grid, that is, its ability to withstand a disturbance and return to an acceptable equilibrium. Although this is stated quite simply, complex modelling, analytical and computational tools are required to analyse, quantify, and improve stability margins.

Given the importance of maintaining stability, a good grounding in this subject is necessary for a power system engineer. In most undergraduate curricula in electrical power engineering, students are generally given only a brief introduction to power system stability in their first course on Electrical Power Systems. This is often their only exposure to power system dynamics and is by itself inadequate to analyse the nuances of the stability problem. Therefore, a follow-up course at the senior undergraduate level or the postgraduate level is necessary. This course should familiarise the student to modelling in the *d-q* domain, which is not only required for machine modelling but also for understanding the vector-control of power electronic converters. Small-signal dynamic analysis and numerical integration methods, which are the necessary tools for analysis of power system dynamic phenomena, should also be covered in such a course.

I think that this book, *Power System Analysis: A Dynamic Perspective*, serves precisely as a bridge between the undergraduate course on power systems and the complex modelling and computational tools that are used for the dynamic analysis of practical systems.

Dr Shubhanga has worked in the area of power system dynamics for more than two decades. He has vast experience in teaching various power system subjects, as well as in the development of laboratory experiments and computer programmes to demonstrate concepts related to power system dynamic phenomena. It is appropriate that he has taken on this task of organising his teaching material in the form of this book.

Therefore, I am delighted to write the foreword for this book. I think the book will be a very useful text and reference book for students of electrical power system engineering.

5th July 2017

A. M. Kulkarni
IIT Bombay

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Preface

Power systems are characterised by a large interconnection of complex dynamic elements such as synchronous machines, transformers, transmission lines, loads, network devices, etc. It is noted that even for a simple one-machine power system, it is not easy to carry out power system analysis without the support of a programming platform. In this regard, this book illustrates various issues related to mathematical modelling of components, interfacing of elements, and assembling of equations to finally solve them efficiently using computational techniques.

The content of the book has evolved as a result of continued teaching of the subject both for undergraduates and postgraduates for a period of more than 15 years. Although many books have been published in this area, it is felt that it is not easy to teach power system dynamics and related topics to students in a class-room environment as the study requires many pre-requisites such as linear systems theory, control theory, numerical methods, basics of power systems, etc. In an attempt to review these areas, many examples and time-domain simulations are presented through dedicated chapters. The main objective of this book is to cover dynamic aspects of power system analysis (in the frequency range below power frequency), giving a thorough insight into their origin and manifestation.

To introduce the system-dynamic aspects, the generalised theory of machines has been presented. A two-coil primitive machine is taken up to illustrate the concept of time-varying parameters in rotating electrical machines.

Both electrical and mechanical equations are given to show the nature of an energy conversion device. These are highlighted both for motor and generator operation, which is generally skipped while introducing the topic. The book presents power systems basics such as sinusoidal generation/operation and control while introducing the phasor-analysis tool as a special case of the state-space analysis. In the second chapter, the state-space modelling of dynamical systems is dealt in a tutorial manner to prepare the students to take-up advanced modelling of power system components. All concepts have been developed from fundamentals so that these topics can be introduced at a lower-semester level.

Before dynamic analysis of a power system is taken up, linear analyses techniques are briefed through many examples including eigenvalue analysis. To appreciate the solution of ordinary differential equations and hence to carry out time-domain simulations, in Chapter 6, a detailed introduction to numerical integration techniques is presented covering most of the popular methods giving their relevance in MATLAB/SIMULINK software. Even the concept of variable-step numerical integration method is discussed. Chapter 7 is dedicated to cover numerical iterative techniques to obtain solution of non-linear algebraic equations. These provide a complete base for handling load flow, fault analysis and complex stability problems.

In order to introduce the synchronous machine modelling and power system analysis with synchronous machines, a reduced order model is derived first, in a systematic manner, before an industry-grade model is discussed. These are covered in Chapters 4 and 5. While deriving the dynamic models, the classical transformation theory such as the Park transformation is neatly developed in Chapter 3. The generality of this transformation in establishing all other transformations found in the literature, is briefed. The application of such transformations in building a phase-lock loop (PLL) is also discussed. Many case studies and examples about synchronous generators are included so that students can easily implement the model and reproduce the results. This motivates self-learning, and students can design new tasks based on the understanding.

In some cases, sample plots are also shown which are acquired in the hardware lab. Having covered the dynamic model of a synchronous machine, the steady-state models are easily derived as a special case of the dynamic model. This, in turn, is directed to initial condition calculation, demonstrating the importance of such a computation in dynamic studies.

In Chapter 8, power system fault analysis is presented with better insight to address system-level studies, giving due importance to computational aspects. This coverage not only helps students to learn about conventional topics, but also provides a logical continuity to model network disturbances in the stability studies both for symmetrical and unsymmetrical faults.

Before introducing different aspects of power system stability analyses, especially related to low frequency power oscillations, a dedicated chapter is included on subsynchronous resonance (SSR) to demonstrate its origin in a simple power system. This tutorial chapter clarifies many issues with power system stability analyses, in general. In the following chapter, i.e., in Chapter 10, the SSR analysis in a slightly complicated the IEEE first benchmark system, is presented. This is found to provide a good introduction to the conventional stability analyses as the power swing-related analysis is much more simpler compared to the SSR analysis.

While introducing the power swing stability analyses in Chapter 12, a thorough insight is provided to a beginner about different types of power system stability, like small-signal stability, transient stability and frequency stability. A mechanical analogy is also given to distinguish these types of stability and its implications on the tools and techniques, and their significance. Many examples are presented to demonstrate the analysis method in simple power systems such as the single-machine connected to infinite bus (SMIB) systems. The effect of generator controllers such as primemover and excitation controllers are clearly explained. All relevant equations are listed, and analysis is carried out using vector and matrix manipulations so that students are driven to develop their own programme codes in MATLAB. The expected sample results are also plotted which can be used for verification. In some cases the captured plots on a lab machine are also shown.

In Chapters 13 and 14, small-signal and transient stability analyses are carried out with interconnected generators. A detailed analytical and time-domain simulation analyses are carried out for a linear multi-mass spring system to illustrate relative and common-mode oscillations and their prediction. These observations are used to get better understanding about power systems' modal behaviour. By taking up a two-machine power system, even frequency stability issues are covered which is generally avoided. In this connection, the concept of centre-of-inertia (COI) variables are also explained. Using these variables, the characteristics of different types of system loads are discussed.

Dynamic models of additional rotating machines such as induction machines, DC machine, and DC-motor driven synchronous machines (MG sets) are discussed in Chapter 15 to understand many hardware experiments conducted in a typical machines lab. The chapter illustrates many examples such as loading test on a synchronous generator, synchronisation of two MG sets, synchronisation to mains, etc., through time-domain simulation of modelling equations.

A major strength of the book is that all fundamental concepts have been explained through simple rigorous mathematical analysis with extensive time-domain simulation results. A number of plots have been given along with necessary elaboration in simple language for easy understanding. For most of the examples and exercises, the associated script and model files developed in MATLAB/SIMULINK will be provided (through website) so that they will be useful for the beginners. A multi-machine power system stability analysis programme (developed in-house) will be made available for free-download (along with user manual) so that many examples in the book can be worked out. These programmes cover both the small-signal and large-signal stability analyses. In the programme, many IEEE standard-type exciters, primemovers and power system stabilisers are implemented. Many speaking tutorials have been generated using this package, which are available on Youtube website which will augment the class-room teaching.

The power system engineering related topics covered in the book can be used as a resource material for teachers, UG/PG/research students, as well as utility personnel.

ACKNOWLEDGMENTS

Prof. A. M. Kulkarni, Department of Electrical Engineering, IIT Bombay, India, has been a constant source of inspiration for me through all the process of the work. His continued support and encouragement guided me to conceive and draft the contents of different chapters. I also thank him for writing the foreword for the book.

Prof. K. R. Padiyar has been my grand-guru.

The Department of Electrical Engineering, National Institute of Technology Karnataka, Surathkal, Mangalore, India, has provided a conducive academic environment for compiling the book. I would like to express my heartfelt thanks to my first Ph.D. student, Dr Shashidhara whose work has been utilised in some chapters. I also thank him for his strong advise to take up book publication rather than going for open-source. Many of my graduate students' project work has contributed to some of the case studies presented in the book. In particular, I wish to thank Surendra, Sreenad, Deepak, E. Prasanthi, Hema Latha, Krishna Rao, and Santosh V. Singh, and an UG student B. Shwetha. In addition, a few students, Gajanana, Rashmi, and Teena, helped me in preparing the manuscript and in drawing figures. The book could not have been completed without the help and support of many friends, colleagues, teachers, staff and the head of the department. I would like to acknowledge the support and encouragements of Prof. K. P. Vittal, Dr G. S. Punekar, Dr Murigendrappa (of Mechanical Engineering Department), Mr. I. R. Rao, and Mr. K. Nagaraja Bhat.

Ms. R. Dheepika, Mr. Sojan Jose and Mr. M. Balakrishnan of M/s. Pearson India Education Services were very helpful throughout the work.

The book would never have been completed without the love and support of my family members and relatives. I would like to place on record the love and moral support of my wife Shobha PS, daughter, Shreya SA, and my mother. I am also grateful to my uncle, Mr. B. Mohana Acharya for his encouragement through out my academic career. I am grateful to almighty for giving me enough strength to complete the book.

K. N. Shubhanga

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About the Author

K. N. Shubhanga has been with the Department of Electrical Engineering, National Institute of Technology Karnataka, Surathkal, Mangalore, India, as a faculty member, for more than 22 years. He received the B.E. degree in Electrical Engineering and the M.Tech. degree in power systems from Mangalore University, India, in 1991 and 1994, respectively, and Ph.D. degree from the Indian Institute of Technology Bombay, in 2003. His research interests are in the areas of FACTS and power system dynamics. He has setup a Power System lab in the department where a scale-down model of a 4-machine power system has been built. This lab has provisions to inter-connect four DC-motor driven synchronous generators so as to illustrate two-area power system operation and control and to acquire real-time dynamic signals of machines. In this lab, UG/PG core course-related experiments are conducted to augment the classroom theoretical concepts. Even in hardware lab such as electrical machines, lab weightages are given to simulation and programming exercises. To support this, he has developed many power system analysis and simulation packages, in-house. Details about the lab setup and other resource materials are available on Google-Drive on the web address: https://goo.gl/l2jnld. Some speaking tutorials (MatSim phase-1) related to power system dynamics are available in the YouTube website.

He is a senior IEEE member and a life member of the IEE (India). He has guided many UG/PG major and minor projects and has involved in research-level activities as well. This effort has resulted in the publication of more than 40 research articles which are indexed in the IEEE digital library. Further, his Ph.D. and PG students have acquired prestigious POSOCO Power Systems Awards in the recent time. He has been teaching subjects such as electrical machines, power systems, power electronic applications to power systems, numerical methods, linear systems, etc. to both UG and PG students. He has presented guest-lectures at various academic institutes and has been a regular reviewer for the IEEE journals and international conferences. He has also held administrative offices both at the institute and at the department levels.

Introduction to Power System Analysis

1

Chapter Outline

In this chapter, we highlight various types of power system analysis that are carried out. In addition, we also discuss the computational issues that are to be handled. The discussion enables the reader to choose appropriate tools and techniques to perform such analysis. A brief introduction to state-space modelling of systems is presented.

In order to prepare a reader to take up synchronous machine modelling in the later chapters, the fundamentals of electromechanical energy conversion process are discussed using illustrative examples which offer time-varying parameters.

1.1 PERFORMANCE OF LARGE POWER SYSTEMS AND THEIR ANALYSIS

A large power system is generally characterised by the following:

1. Large power transfer over large distance from generation centres to load centres.
2. Lack of power flow controllability due to heavy AC connection where power flow in various lines gets decided by physical AC laws. Hence, transmission systems are unevenly utilised.
3. Heavy AC interconnection with limited high-voltage direct current (HVDC) links between control areas. This leads to unscheduled power flow.
4. Due to limited right-of-way, it is not easy to take up new expansion planning to augment the system.

5. Stability issues largely constrain power transfer. This may require large load margins for equipment, making the system bulky and non economical.
6. While transferring large power over long distance, voltage control becomes a major control problem.
7. The system possesses very poor damping for electromechanical oscillations, thus restricting the loading level.

In order to develop an understanding of such a system, it is necessary to carry out a detailed power system analysis under steady-state and dynamic conditions. This generally involves the following types of analysis:

1. **Reliability analysis:** It involves a probabilistic study and denotes the ability to supply load adequately with as few interruptions as possible. It is a function of time-average performance of the power system over a long period [1].
2. **Security analysis:** It involves the determination of the ability of the power system to meet the load demand without any violation of apparatus-operating limits against possible contingencies. It is to be noted that for the system to be reliable, it must be secure most of the time. Security and stability issues are strongly coupled.
3. **Stability analysis:** It determines the ability of the system to remain intact without losing synchronous operation. For a system to be secure, it must be stable. Security and stability are time-varying attributes and are functions of the operating state and a candidate contingency. Therefore, when security issues are analysed, it is customary to make a distinction between security analysis carried out at an operating point, referred to as *steady-state security*, and those studies in which transition to a new operating condition, is analysed, referred to as *dynamic security analysis* (DSA) [2].

1.2 CLASSIFICATION OF POWER SYSTEM STABILITY PROBLEMS

It is well-known that stability of linear systems is neither dependent on the operating point nor on the forcing function, whereas for non-linear systems, like power systems, the stability of the system is decided by the equilibrium point at which it is operating and the magnitude of the forcing function. Therefore, as per [3, 5], a classification is done to perform power system stability studies as shown in Table 1.1 based on the time-scale and driving force of instability.

Some features of the tabulated-type of power systems stability are briefly discussed below [4].

Table 1.1 Classification of Power System Stability Analysis.

Time range	Generator-driven (Rotor-angle stability)		Load-driven
Short-term	Small-signal stability	Transient stability	Voltage stability
Long-term	Frequency stability		

1. **Rotor-angle stability:** This analysis is mainly concerned with electromechanical oscillations and large angle variation of the generator rotors. Small-signal stability analysis deals with the operating point stability with respect to low-frequency oscillations which can range from 0.1–3 Hz. This stability is the fundamental requirement of any system. In case of transient stability analysis, the system performance is studied for large disturbances. In this short-term time frame, there is no common frequency throughout the system. However, in the long-term time scale when the short-term dynamics are stable, the frequency excursions gain importance since both under- and over-frequency system operations are detrimental to system integrity [6]. This long-term time scale, which typically lasts for several minutes, necessitates modelling of slow dynamical devices such as tap changers, prime-movers controllers, boilers and their controls, etc. Such long-term dynamics largely depend on the generation-load imbalance irrespective of network connectivity within each connected area.

   In addition to the above listed synchronous generator rotor-related stability performances, another type of small-signal stability analysis that is studied is the subsynchronous resonance (SSR) problem. This analysis involves modelling of combined steam turbine-generator mechanical systems, including the dynamics of capacitor-compensated electrical transmission network. Here, the frequency of interest lies in the range of 15–45 Hz, i.e., subsynchronous range [7, 8]. With the integration of induction generator- and doubly-fed induction generator-based wind energy conversion systems into the conventional AC grid, it has led to another type of stability problem, referred to as *speed instability* [9].
2. **Load-driven stability:** In contrast to angle stability where we are concerned about *relative* and *common* angle oscillations, here, the analysis mainly involves determination of voltage-maintaining-ability of power systems. This analysis requires full network representation along with modelling of dynamic load components (induction motors, electronically controlled loads, HVDC interconnections), voltage control devices such as exciters, on-load tap-changers' controllers, any reactive power sources, and so on. The time scale of analysis and complexity of modelling is often similar to the short-term rotor-angle stability analysis. Sometimes, in long-term voltage stability studies, for simpler and faster analysis, the possibility of voltage instability is understood by detecting the absence of equilibrium points using a static tool like power flow programmes, even if the voltage stability problem is inherently dynamic in nature.

It should be noted that it is not easy to carry out power system stability analysis without performing a categorisation of stability problems since such a classification facilitates the following:

1. To choose appropriate degree of details for system representation: Time frame of study and time response of components helps us make appropriate approximations in any analysis. For example, while investigating the impact of the control set point of boiler, it is not necessary to solve the complex transmission line wave equations. This is obvious because the boiler response time is in the range of minutes, whereas the wave travel time is in the order of milliseconds. Such *engineering approximation* is very useful in system studies [10].
2. To select an appropriate analytical tool: For example, when small-signal stability analysis is carried out, the non-linear system equations are linearised around an operating point whose stability performance is to be investigated. The linear control theory is then applied, assuming that the system trajectories do not trigger non-linear behaviour. Therefore, one may just perform the eigenvalue analysis and predict the stability without solving the system's differential-algebraic

equations (DAEs). If it is required to obtain the large-signal performance, one is left with no choice but to carry out numerical solution of non-linear DAEs.

3. To identify key factors that contribute to instability and to devise methods for improving stable operation.

1.3 POWER SYSTEM STABILITY ANALYSIS—COMPUTATIONAL ISSUES

To perform power system stability analysis, a knowledge of the following topics is desirable.

1. Linear algebra: The theory of vector spaces, basis, the theory of matrices, similarity transformations, time-variant and time-invariant systems, and so on [12].
2. Systems theory: Identification of linear and non-linear systems, state-space modelling, etc [11].
3. Computational techniques: This includes the following system of equations:
   (a) Algebraic system of equations:
      (i) Linear algebraic system of equations: Generally, these equations will be in the form, $A\underline{x} = \underline{b}$. Solution techniques may employ either elimination techniques such as Gauss or decomposition methods such as $[L][U]$, $[L][L]^T$, and so on [13]. In power system analysis, we come across this form of equations in tasks such as (i) steady-state fault analysis, (ii) in each iteration of power flow analysis where deviations are calculated, (iii) in each time step when implicit numerical integration technique is employed to solve linear differential equations, etc.
      (ii) Non-linear algebraic system of equations: Generally, these equations will be in the form, $F(\underline{x}) = \underline{0}$. For such systems, only numerical solution is possible, and solutions are obtained by using *numerical iteration techniques*. Solution techniques may be in form of fixed point iteration (also referred to as Gauss method), Gauss-Seidel, or Newton-based methods. A classical power system analysis task where we use this type equations is power flow analysis.
   (b) Differential system of equations: The following are the two major kinds of differential equations depending on the nature of equations:
      (i) Linear differential equations: Here, differential equations govern the dynamics of linear systems. Linear systems are those which contain linear elements which exhibit a linear relationship between its excitation and response. Such system of differential equations is amenable for closed form of *analytical solutions*. They need not be solved using numerical integration techniques. Such differential equations are easily recognisable as linear if they are written in *state-space* form [14]. For example, in an RL-series circuit, if the inductor is air-core type, its inductance value remains constant independent of flux, hence the current. In other words, the flux-current relationship is linear, resulting in a constant inductance or we state that the inductor remains *unsaturated*. Therefore, the differential equation governing the dynamics of such a circuit remains linear.
      (ii) Non-linear differential equations: Here, differential equations govern the dynamics of non-linear systems. In the above example of a series RL circuit, if the inductor saturates, we can seen that the flux-current relationship is not linear. This leads to a non-linear differential equation. An important difficulty in solving such differential equations is that

their solution cannot be obtained analytically, or in other words, the solution cannot be written in a closed form of expression. The solution values of variables can only be determined at discrete time-instants by applying numerical integration techniques. Therefore, through a *numerical* processing, the sampled-version of the solution flow is obtained.

(c) Differential-algebraic equations (DAEs): Here, coupled differential and algebraic equations are solved *simultaneously*. Such requirement arises when systems such as power systems, weather systems, chemical systems, and so on, are modelled. The following approaches may be employed [10]:

  (i) Partitioned solution approach: Here, the differential and algebraic equations are solved separately in an alternate fashion. To solve differential equations, one may employ either explicit or implicit technique.

  (ii) Simultaneous implicit solution approach: In this case, the differential equations are *algebraiced* using an implicit integration technique and are augmented to the algebraic equations. The resulting algebraic equations are collectively solved using Newton-based iterative schemes.

  The above methods will be further briefed in later sections.

(d) Evaluation of eigenvalues and eigenvectors: It is known that eigenvalue analysis is one of the widely followed methods to understand the operating point stability of power systems. In addition, eigenvectors facilitate easy prediction of dominance of a frequency component in a state-variable, thus guiding the placement and tuning of controllers. The eigenvalues are the roots of the characteristic polynomial of a system. It is well-understood that beyond the fourth order, it is not easy to determine the roots of the polynomial. Hence, one is forced to use numerical techniques to estimate eigenvalues and eigenvectors. In this regard, QR technique [13] and modified Arnoldi methods (MAM) [15] are more widely used.

1.4 POWER SYSTEM STABILITY—MODELLING ISSUES

Developing an accurate mathematical model for a system and its constituent components is an important step in power system analysis. A model is said to be suitable if it can faithfully reproduce the real-world observations adequately, i.e., to a desired level of human satisfaction, in the computational domain. A mathematical model development for a component starts by applying known physical laws which govern the operation of the component. However, the final usage of a model for a component depends on some of the following factors:

1. The possibility of determining the parameters involved in the model.
2. The human effort required to implement the model.
3. The availability of a suitable computational tool.
4. The computational effort to obtain the solution.
5. The time-response of the performance desired.
6. The computational accuracy required.

Therefore, there is always a trade-off between all these factors, which decide to what detailed level a component model is used in system analysis. In any such studies, suitable *engineering approximations* are made to obtain a response (in the mathematical domain) which *adequately* match the real-world physical response. A good example for demonstrating the importance of approximations in modelling is the methodology employed for modelling of synchronous machines. To support this, Prof. M. A. Pai acknowledges in his book [10] by stating 'There is probably more literature on synchronous machines than on any other device in electrical engineering. Unfortunately, this vast amount of material often makes the subject complex and confusing.'

As per literature, it is noted that no experimental test can be conducted on a synchronous machine in the laboratory to determine the preliminary parameters desired by the basic model. In such a case, one is required to obtain hybrid parameters of a machine. These parameters are commonly referred to as *the standard parameters* of a synchronous machine. To make the basic model usable, it is modified in terms of the standard parameters. Therefore, obtainability or availability of machine data often constrains one from using a detailed model for the synchronous machine. In some cases, depending on the acceptable level of accuracy, we can use reduced order model as well.

In power system analysis, the following two modelling approaches are generally employed:

1. Distributed parameter representation: Here, time and space are treated as independent variables. For example, while modelling high-frequency transients such as lightning or switching surges, it is desirable to consider the spacial distribution of basic parameters R, L, and C because, in this case, the length of the line is comparable to the electromagnetic wave-length [16] leading to considerable wave-travel time. While modelling such components, we invariably use partial differential equations (PDE) and they require dedicated numerical methods to capture the wave-travel effect. Bergeron's method [10] is one such method employed in electromagnetic transient packages such as PSCAD/EMTDC and EMTP RV [17].
2. Lumped parameter representation: Here, only time is treated as an independent variable. The distributed influence of the basic effects, such as resistance, inductance and capacitance, are assumed to be lumped at a place leading to a circuit model for the component. This enables us to write ordinary differential equations (ODEs) to describe their dynamics as the wave-travel time is negligible. The power system stability analysis uses such representation for components.

1.4.1 Modelling of Dynamical Systems

Generally, there are two kinds of modelling approaches:

1. **System function-based approach:** In this approach, a given ODE is *algebraiced* and written in the form of a function involving the ratio of polynomials. Such representation is applicable only to linear systems. While algebraicing, the initial values on the variables are set to zero. This representation is generally employed for ease of depiction of dynamics of a system. A typical system function is given by

$$\frac{X(s)}{U(s)} = \frac{Np(s)}{Dp(s)}$$

where s is an operator which represents $\frac{d}{dt}$ with zero initial conditions.

For example, a simple single-time constant delay circuit is depicted as

$$H(s) = \frac{X(s)}{U(s)} = \frac{1}{(1+sT)} \tag{1.1}$$

where T is in seconds.

2. **State-space-based approach:** In this approach, a given nth order differential equation is written as a set of n numbers of first order differential equations by appropriately choosing a *state-variable* vector [12]. An important advantage of this approach is that this representation can be used for either linear or non-linear systems with desired initial value on states.

1.4.1.1 State-space Modelling of a System

Every physical system is causal. Causal or non-anticipatory system is one in which the present output depends on the past and the present inputs and not on future inputs. Defining a *state* for such a system overcomes the problem of tracking the input $u(t)$ from $t = -\infty$, for determining the present output. The state x of a system is that variable whose value is known at time t_0 is the information at $t = t_0$ that, along with the input, for $t \geq t_0$, *uniquely* determines the output $y(t)$ for all $t \geq t_0$. Here, x is referred to as a *state-variable* and it possess the following properties:

1. State-variables are continuous in time, i.e., $x(t_0^-) = x(t_0^+)$.
2. The selection of state-variables is not unique.
3. The number of state-variables chosen for a system indicates the *degrees-of-freedom* associated with the system. It also represents the *order* of a system.

Let us examine another classification of lumped systems which is given below [18]:

1. Time-invariant systems: If the initial state and the input are the same, no matter at what time they are applied, the output waveform will be the same. For such systems, the parameters of the systems are constant and independent of time.
2. Time-varying systems: Here, parameters of the systems are not constants and are functions of time. Examples are change of parameters due to aging, primitive inductance of rotating machines, etc.

Note: Most physical systems are modelled as *time-invariant* systems as it is not easy to handle time-varying systems in terms of solution of systems of equations.

The state-space description of a lumped linear time-invariant (LTI) systems is given by

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{u} \\ \underline{y} &= C\underline{x} + D\underline{u} \end{aligned} \tag{1.2}$$

For a p-input and q-output system, $\underline{u}$ is $p \times 1$ vector and $\underline{y}$ is a $q \times 1$ vector. If the system has n state-variables, then $\underline{x}$ is an $n \times 1$ vector.

- $A \rightarrow n \times n$: state-feedback matrix
- $B \rightarrow n \times p$: input matrix
- $C \rightarrow q \times n$: output matrix
- $D \rightarrow q \times p$: direct-feed matrix

It should be noted that the A matrix is constant and is independent of time for LTI systems. Also, note that the state-space description involves a set of n first-order linear differential equations and q algebraic equations.

The above description of the state-space model, i.e., in the matrix notation see eq. (1.2), is feasible only for linear systems. For non-linear systems, the state model cannot be written in *matrix* form. Then, it can only be written in the following form:

$$\dot{\underline{x}} = F(\underline{x}, \underline{u}, t) \tag{1.3}$$

where $\underline{x}$ denotes the stat-variable vector, $\underline{u}$ -represents the input vector.

If the time derivative of the state is not an explicit function of time *t*, such a system is referred to as an *autonomous system*. As stated earlier, for non-linear systems, the solution cannot be obtained by analytical means. Only a numerical solution is possible. It is also important to note that to apply any numerical integration technique, the system should be modelled only in state-space form.

Example 1.1

For the system function shown in eq. (1.1), a state-space model can be written as

$$\dot{x} = -\frac{1}{T}x + \frac{1}{T}u(t) \tag{1.4}$$

For a unit-step input, the zero-state response is given by

$$y(t) = x(t) = 1 - e^{\left(-\frac{t}{T}\right)}$$

Let $z = \frac{1}{T}x$, then $\dot{x} = T\dot{z}$. Making this substitution in eq. (1.4) we get,

$$\dot{z} = -\frac{1}{T}z + \frac{1}{T^2}u(t) \tag{1.5}$$

and the desired output is given by

$$x(t) = Tz(t) \tag{1.6}$$

From eq. (1.5), if the system function is obtained, then we get

$$H_1(s) = \frac{Z(s)}{U(s)} = \frac{\frac{1}{T}}{(1+sT)}$$

Now using eq. (1.6) in the above equation, we get

$$\frac{\frac{1}{T}X(s)}{U(s)} = \frac{\frac{1}{T}}{(1+sT)} \tag{1.7}$$

The above equation, clearly demonstrates that for a given system function there is no unique state-model. This is because $H(s)$ given in eq. (1.1) can be obtained from either eq. (1.4) or eq. (1.5).

Example 1.2

Consider another system function given by

$$H(s) = \frac{X(s)}{U(s)} = \frac{sT}{(1+sT)} \tag{1.8}$$

In the above case, if we simply cross-multiply LHS and RHS terms to get a state-model, then it results in taking a derivative of the input u which should be strictly avoided in building a state-model. A procedure to avoid this is given below.

The above function is rewritten as

$$X(s) = \frac{sT \times U(s)}{(1+sT)}$$
$$= sT \times Z(s) \tag{1.9}$$

where

$$Z(s) = \frac{U(s)}{(1+sT)}$$

A state-model for the above system function can be easily written as

$$\dot{z} = -\frac{1}{T}z + \frac{1}{T}u(t) \tag{1.10}$$

From eq. (1.9), the output equation can be written as

$$x(t) = T\dot{z} \tag{1.11}$$

Using eq. (1.10) in the above equation, we get the desired output equation as

$$x(t) = -z + u(t) \tag{1.12}$$

Therefore, for the system function given in eq. (1.8), the complete state-model is given by eqs. (1.10) and (1.12).

1.5 ELECTROMECHANICAL ENERGY CONVERSION

The process of electromechanical energy conversion involves two forms of energy conversion:

1. Conversion from electrical to mechanical which is referred to as motor operation.
2. Conversion from mechanical to electrical which is referred to as generator operation.

In both the processes, there exists a magnetic field which acts as a media between electrical and mechanical systems. This media should be *filled* with energy to facilitate the conversion[1]. This implies that electrical circuit is dominantly inductive, and its inductance may be time-varying. To understand this complex energy exchange and interaction, a few examples are presented [19, 20].

1.5.1 Behaviour of an Inductor with Fixed Armature Core

Consider a coil as shown in Figure 1.1, where the armature core is fixed and is assumed to be unsaturated.

Applying Kirchhoff's Voltage Law (KVL), we can write the circuit equation as

$$v = Ri + \frac{d\psi}{dt} \tag{1.13}$$

[1]This clarifies the concept of reactive power loss at fundamental frequency under sinor steady-state. An analogy to this concept is the water in a pipe. In order to have water discharge at a constant rate at the other end of the pipe, the pipe should be filled with water all the time as long as we need water delivery. This amount of water is not available for utilisation. This is equivalent to a loss. This represents the reactive power and the water discharge at a given rate denotes the real power transferred.